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Absence of Interaction in Lie Field Theories*

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Abstract. It is shown that theories of relativistic Lie fields cannot lead to scattering or reactions, even if an infinite number of Lie fields is present.

1. Introduction

Generalized free fields can be considered to be the lowest order case of models of fields whose commutator can be expanded in products of Heisenberg fields containing up to a given number of factors. The next more complicated case, in which the commutator is a linear functional of the field, shares with the case of generalized free fields the property of being a soluble model, but has a more complicated structure, which might lead to scattering and reactions. For this reason, we suggested study of this case, now called Lie fields, and pointed out that the vacuum expectation values are determined by recursion in terms of the distributions introduced in the ansatz for the commutator and thus determine the field. At the same time, we pointed out that translation invariance, Lorentz invariance, locality, and positive energy spectrum can easily be satisfied by themselves, but that the Jacobi identity for the commutator and the positive definiteness conditions for the vacuum expectation values, both of which are essential for a consistent model of field theory, seemed to be difficult to fulfill [1]. Recently, LOWENSTEIN [2] has shown that the case of Lie fields is not empty by giving non-trivial examples of scalar Lie fields. These examples make it worthwhile to study further the possibility that Lie field theories give rise to scattering and reactions. GLASER [3] has shown that a Lie field theory with a finite number of fields gives an elastic scattering amplitude whose absorptive part has only a finite number of partial waves in the s-channel. Such an absorptive part is a polynomial in $z = \cos\theta$ and fails to have the singularities in z which are required by unitarity and crossing [4]. In this article we will show that no scattering or reactions can occur in a Lie field theory, even if an infinite number of Lie fields is present. Our argument is based on unitarity, as expressed by the commutation relations of the in and out

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