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On Equal-Time Commutation Relations of Renormalized Currents in Perturbation Theory I

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Abstract. Equal-time current commutation relations are considered in renormalizable field theories. Renormalized currents are obtained by means of solutions of the Yang-Feldman equations for Heisenberg field operators in perturbation theory. For the computation of matrix elements of current commutators we apply Jost-Lehmann-Dyson type techniques. The equal time limit is taken with the help of symmetrical time-smearing functions which interpolate the δ-function. Our methods avoid any cut-off procedure and lead therefore to unambiguous results. In order to avoid spin complications, our general methods are applied to trilinear resp. quadrilinear couplings of isoscalar and isovector spin 0-mesons in first order perturbation theory. We find that the zero-space components of the current-commutator matrix elements behave for small time separation T like $\ln(T)$ $\mathrm{grad}_x \delta(\boldsymbol{x} - \boldsymbol{y})$.

Introduction

It was pointed out by the present authors some time ago that the concept of equal-time current commutation relations (ETCR) is compatible with the general principles of quantum field theory [1]. In order to understand the dynamical content of ETCR and their general form allowed within the field theoretic framework it may be helpful to discuss ETCR for renormalizable field theories in perturbation theory. In this paper we take up the discussion of this problem.

The first investigations along this line have been undertaken by JOHNSON and LOW [2] and other authors [3]. The procedure used in these papers we want to criticize for two reasons:

1. No explicitly renormalized currents are used.
2. The ETCR $[j_\mu(x), j_\nu(y)]_{x_0 = y_0}$ is computed by taking appropriate time limits from the time-ordered product $T[j_\mu(x), j_\nu(y)]$.

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