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Representations of Anticommutation Relations and Bogolioubov Transformations

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Abstract. A description of the quasi-free states on a Clifford algebra and their representations is given, and we prove that the pure quasi-free states are Fock States.

I. Introduction

In this paper we complete the study of quasi-free states on a Clifford algebra started in ref. [1], where essentially the translation invariant states were treated. Here we use however a different method which turned out to be more powerful to derive general properties of the set of quasi-free states. The relation with ref. [1] is established in appendix A.

Our starting point is a C^*-Clifford algebra $\overline{\mathfrak{A}(H, s)}$ built on an euclidean space (H, s) (i. e. H is a real vector space on which a bilinear, symmetric, positive definite form s is defined). Without loss of generality we suppose that H is complete. For more details we refer to ref. [2]. Let B be the canonical mapping of H into $\overline{\mathfrak{A}(H, s)}$ such that

$$[B(\psi), B(\varphi)]_+ = 2s(\psi, \varphi) \quad \text{for} \quad \psi, \varphi \in H . \tag{1}$$

Let T be an element of the group $\mathcal{O}(H, s)$ of orthogonal operators on (H, s) and $\alpha(\overline{\mathfrak{A}(H, s)})$ the group of automorphisms of $\overline{\mathfrak{A}(H, s)}$, then the mapping $B(\psi) \to B(T\psi)$ can be extended to an automorphism τ_T of $\overline{\mathfrak{A}(H, s)}$. Furthermore the operator $\tau : T \to \tau_T \in \alpha(\overline{\mathfrak{A}(H, s)})$ is a monomorphism. In theorem 1 we prove that any two Fock states are related by such an automorphism. We also remark that such an automorphism corresponds to a generalized Bogoliubov transformation (see appendix A).

Furthermore we explicitly construct all representations induced by quasi-free states and give a criterium under which they are irreducible.

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