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Eine scheinbare Abschwächung der Lokalitätsbedingung. II

H.-J. Borchers
Institut für Theoretische Physik der Universität Göttingen

K. Pohlmeyer
II. Institut für Theoretische Physik der Universität Hamburg

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Abstract. If for a relativistic field theory the expectation values of the commutator $(\Psi|\,[A(x), A(y)]\,|\Psi)$ vanish in space-like direction like $\exp\{-\mathrm{const}\,|(x-y)^2|^{\alpha/2}\}$ with $\alpha > 1$ for sufficiently many vectors Ψ, it follows that $A(x)$ is a local field. Or more precisely:

For a hermitean, scalar, tempered field $A(x)$ the locality axiom can be replaced by the following conditions

1. For any natural number n there exist a) a configuration $X(n)$:

$$X_1, \ldots, X_{n-1} \qquad X_1^i = \cdots = X_{n-1}^i = 0 \qquad i = 0,3$$

with $\left[\sum_{i=1}^{n-2} \lambda_i (X_i^1 - X_{i+1}^1)\right]^2 + \left[\sum_{i=1}^{n-2} \lambda_i (X_i^2 - X_{i+1}^2)\right]^2 > 0$ for all $\lambda_i \geqq 0$ $i = 1, \ldots, n-2$, $\sum_{i=1}^{n-2} \lambda_i > 0$, b) neighbourhoods of the X_i's: $U_i(X_i) \subset R^4$ $i = 1, \ldots, n-1$ (in the euclidean topology of R^4) and c) a real number $\alpha > 1$ such that for all points $(x): x_1, \ldots, x_{n-1}: x_i \in U_i(X_i)$ there are positive constants $C^{(n)}\{(x)\}$, $h^{(n)}\{(x)\}$ with:

$$|\langle [A(x_1) \ldots A(x_{n-1}), A(x_n)] \rangle| < C^{(n)}\{(x)\} \exp\{-h^{(n)}\{(x)\} r^\alpha\} \quad \text{for } x_n = \begin{pmatrix} 0 \\ 0 \\ 0 \\ r \end{pmatrix}, \; r > 1\,.$$

2. For any natural number n there exist a) a configuration $Y(n)$:

$$Y_2, Y_3, \ldots, Y_n \qquad Y_3^i = \cdots = Y_n^i = 0 \qquad i = 0,3$$

with $\left[\sum_{i=3}^{n-1} \mu_i (Y_i^1 - Y_{i+1}^1)\right]^2 + \left[\sum_{i=3}^{n-1} \mu_i (Y_i^2 - Y_{i+1}^2)\right]^2 > 0$ for all $\mu_i \geqq 0$, $i = 3, \ldots, n-1$, $\sum_{i=3}^{n-1} \mu_i > 0$, b) neighbourhoods of the Y_i's: $V_i(Y_i) \subset R^4$ $i = 2, \ldots, n$ (in the euclidean topology of R^4) and c) a real number $\beta > 1$ such that for all points $(y): y_2, \ldots, y_n$ $y_i \in V_i(Y_i)$ there are positive constants $C_{(n)}\{(y)\}$, $h_{(n)}\{(y)\}$ and a real number $\gamma_{(n)}\{(y)\} \in a$ closed subset of $R - \{0\} - \{1\}$ with: