Statistical Mechanics of Quantum Spin Systems. II*

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Abstract. In the first part of this paper we continue the general analysis of quantum spin systems. It is demonstrated, for a large class of interactions, that time-translations form a group of automorphisms of the C^*-algebra $\mathfrak{A}$ of quasi-local observables and that the thermodynamic equilibrium states are invariant under this group. Further it is shown that the equilibrium states possess the Kubo-Martin-Schwinger analyticity and boundary condition properties. In the second part of the paper we give a general analysis of states which are invariant under space and time translations and also satisfy the KMS boundary condition. A discussion of these latter conditions and their connection with the decomposition of invariant states into ergodic states is given. Various properties pertinent to this discussion are derived.

1. Introduction

In a previous paper [1], hereafter referred to as I, the general analysis of the statistical mechanics of quantum spin system was begun. The primary purpose of the present paper is to continue this analysis. We focus our attention on the problems involved with the time-development of such a system in equilibrium. Our analysis begins with a proof that time-translations form a one-parameter group of automorphisms of the C^*-algebra $\mathfrak{A}$ of quasi-local observables. It is then possible to demonstrate that the single phase equilibrium states, whose existence was established in I, are both invariant under time translations and satisfy the Kubo-Martin-Schwinger analyticity and boundary properties.

Recently Haag, Hugenholtz, and Winnink [2], have studied the properties of states invariant under time translations satisfying the KMS boundary condition and have derived a number of properties of such states which are independent of spatial structure. In the second half of this paper we give an analysis of some of the implications of local structure and invariance under space translations. The KMS boundary condition plays somewhat the role of a spectrum condition and allows us to derive properties whose analog in relativistic field theory is dependent on the spectrum condition. In particular it is shown that time translations leave the center of the covariance algebra, generated by $\mathfrak{A}$ and the group

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