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On the Continuity of Automorphic Representations of Groups

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Abstract. Given a weakly continuous automorphic representation α of a group G on a concrete C^*-algebra $\mathfrak{A}$, we show that a mild joint continuity condition makes it possible to extend α to a weakly continuous representation of G on the weak closure of $\mathfrak{A}$. If G is locally compact and $\mathfrak{A}$ is a von Neumann algebra, this condition is automatically satisfied.

Introduction

Let G be a topological group, and let $\mathfrak{A}$ be a C^*-algebra with the identity operating on a Hilbert-space H. Let $\alpha: G \to \operatorname{aut}(\mathfrak{A})$ = the group of $*$-automorphisms of $\mathfrak{A}$, be a homomorphism, and let α_g denote image of $g \in G$ under α. We require that the map

$$A: (g, x) \to \alpha_g(x); \qquad g \in G, \qquad x \in \mathfrak{A}$$

of $G \times \mathfrak{A}$ into $\mathfrak{A}$ is *separately continuous* when restricted to $G \times \mathfrak{A}_1$, and $\mathfrak{A}_1$ = the unit sphere of $\mathfrak{A}$, is given the weak operator topology. In this case we say that α is an *automorphic representation* of G on $\mathfrak{A}$.

Since $(\alpha_g)^{-1} = \alpha_{g^{-1}}$, it follows in particular that each α_g; $g \in G$, is weakly bi-continuous on the unit sphere of $\mathfrak{A}$, and therefore has an extension to $\overline{\mathfrak{A}}$ = the weak closure of $\mathfrak{A}$ on H, as a $*$-automorphism which we again denote by α_g ([4], remark 2.2.3). Each α_g is weakly bi-continuous on the unit sphere of $\overline{\mathfrak{A}}$, and the map $\bar{\alpha}: G \to \operatorname{aut}(\overline{\mathfrak{A}})$ obtained this way is a homomorphism. It is a question of some interest to know under what conditions $\bar{\alpha}$ is an automorphic representation of G on $\overline{\mathfrak{A}}$, i.e., when the map $g \to \alpha_g(x)$; $x \in \overline{\mathfrak{A}}_1$, is weakly continuous. For a motivation, see f.i. DELL'ANTONIO [2] or KADISON [5]. It turns out that a mild *joint continuity* condition on the map A is sufficient, and that this condition is automatically satisfied if $\mathfrak{A}$ is a von Neumann algebra.

I. Extension of Automorphic Representations

We say that the automorphic representation α of G on $\mathfrak{A}$ is *extendable*, of $\bar{\alpha}$ is an automorphic representation of G on $\overline{\mathfrak{A}}$. Clearly, several other topologies on $\mathfrak{A}$ were possible in the definition of an automorphic rep-

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