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Quantum Field Model with Unrenormalizable Interaction

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Abstract. The unitary relativistic model of quantum field theory with rapidly increasing spectral function (i.e. it grows faster than any finite power of momentum) is investigated. It is shown that there exist nontrivial Lagrangians, leading to this kind of spectral functions and allowing to construct the local theory without the ultraviolet divergences on their basis. In this theory the S-matrix is unitary and not e qual identically to unity.

1. Introduction

The problem arising through the attempts to construct finite unrenormalizable theory, as well as through ascertaining the connection among the nonlocal theories and the unrenormalizable theories have attracted the attention of many authors [1—13, 20]. Some years ago one supposed that the unrenormalizable theories were nonlocal. But recently one discovered that there exist some region where the field theories with rapidly increasing spectral functions were local [12—13]. Also at the same time construction of the finite unrenormalizable field theories was attempted at [1, 6, 7, 9]. At present there are yet many uncertainties in these questions.

Due to the big complication of these problems it is interesting to consider a simple model in order to explain some general properties of the unrenormalizable theories.

We investigate here a model of the quantum field theory with the Lagrangian [14—16]

$$L(x) = L_0(x) + L_{\text{int}}(x)\,, \tag{1.1}$$

where $L_0(x)$ is the Lagrangian of the free fields and

$$L_{\text{int}}(x) = -g :\bar{\psi}(x)\,\tau_1\,\gamma_\nu\,\psi(x)\,\partial_\nu\,\varphi(x): \;-\; \Delta m :\bar{\psi}(x)\,\tau_3\,\psi(x): \;. \tag{1.2}$$

Here τ_1 and τ_3 are the isotopic spin matrices, γ_ν are the Dirac matrices, $\psi(x)$ is the spinor field operator, and $\varphi(x)$ is the scalar field operator.

The Lagrangian (1.1) has the following remarkable property: when $\Delta m = 0$, it is reduced to the diagonal form

$$L_{\Delta m = 0}(x) = L_0(\psi'(x),\, \varphi(x)) \tag{1.3}$$

by means of the unitary transformation

$$\psi'(x) = \psi(x)\exp\{ig\tau_1\,\varphi(x)\}\,. \tag{1.4}$$