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On the Extension of Invariant Partial States in Statistical Mechanics

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Abstract. We consider a class of physical systems often encountered in such theories as statistical mechanics, namely those which admit an amenable group as a group of symmetries. We first establish the existence of invariant state(s) with respect to the symmetry group considered. We next introduce the notion of (extremal) potentially invariant states for an reduced description of the physical system considered; we then show that every such state can be extended to an (extremal) invariant state corresponding to a complete description.

1. Introduction

Segal's original proposition [1] to discuss in algebraic terms the structure of physical theories has been considered with great details in the recent years with the hope to bring at least some degree of order in the confusing puzzles offered to physicists by theories dealing with infinite systems, such as quantum field theory [2] or statistical mechanics [3].

Besides its formal achievements, the C^*-algebraic approach has been successfully tested in statistical mechanics on some models as simple as the ideal Bose- [4] and Fermi-gas [5]. Some success have also been registered when dealing with some very particular types of interacting systems, the most typical of which is the BCS-model [6]. These interacting models can be characterized in mathematical terms by saying that their equilibrium states are 'quasi-free states' [7]. In the every day language of the physicist this means that some kind of 'mean free field' approximation [9] (generalizing to other situations the Weiss' discussion of ferromagnets) becomes exact in the thermodynamical limit. However the 'mean free fields'-models are well known [10] to suffer from serious draw-backs in the general case. In particular, the behaviour of some of the thermodynamical functions is predicted quite wrongly by such theories. Consequently the next step, that the algebraic approach has to

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