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Analytic Continuation of Group Representations. VI*

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Abstract. The Gell-Mann formula for analytically continuing group representations is worked out explicitly for more cases than in previous work, and extended to certain pseudo-Riemannian symmetric spaces. The method of finding the asymptotic behavior of matrix elements of group representations introduced in Part V is developed in more detail and it is shown how it leads to new mathematical problems in the theory of dynamical systems and Hilbert space theory.

I. Introduction

We continue work on the order of ideas introduced in the earlier papers of this series [4]. The main types discussed here are: Further development of the Gell-Mann formula [3], and development of the theory of asymptotic behavior of matrix elements of group representations.

II. The Gell-Mann Formula in Terms of the Enveloping Algebra

Suppose that $\mathbf{K}$ is a Lie algebra, with a basis $Z_i (1 \leqq i, j, \ldots, \leqq n$; summation convention) such that:

$$[Z_i, Z_j] = c_{ijk} Z_k .$$

Suppose $\mathbf{P}$ is an abelian Lie algebra, with a basis X_a $(1 \leqq a, b, \ldots \leqq m)$. Suppose that $\mathfrak{G}' = \mathbf{K} + \mathbf{P}$ is a Lie algebra with $\mathbf{P}$ an ideal, i. e.,

$$[Z_i, X_a] = c_{iab} X_b .$$

Form the elements:

$$X_a^\lambda = [\Delta, X_a] + \lambda X_a$$

of $U(\mathfrak{G}')$, the universal enveloping algebra of $\mathfrak{G}'$ (Δ is the second order Casimir operator of $\mathbf{K}$). In [3] we have investigated the condition that $[X_a^\lambda, X_b^\lambda]$ be expressible in terms of the Z's, where $\mathfrak{G}$ is realized as a Lie algebra of skew-Hermitian operators on a Hilbert space H. Here, we will present a representation-independent version of this calculation, aiming to find conditions that $[X_a^\lambda, X_b^\lambda]$ be expressible within the en-

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