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On the Factor Type of Equilibrium States in Quantum Statistical Mechanics

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Abstract. A theorem is derived giving sufficient conditions for a factor to be either finite or purely infinite. These conditions are:

i. In the Hilbert space $\mathfrak{H}$ exists a conjugation operator J transforming the factor $\mathfrak{R}$ into its commutant $\mathfrak{R}'$.

ii. There exists a one parameter abelian group of automorphisms of $\mathfrak{R}$ implemented by unitary operators U_t weakly continuous in t and commuting with J.

iii. There is a cyclic and separating vector Ω, which is invariant for J and which is the only vector in $\mathfrak{H}$ invariant for U_t.

This theorem is of interest for Statistical Mechanics since representations of thermal equilibrium states satisfy these conditions [1]. One finds that the representations of equilibrium states corresponding to one phase are factors of type III.

1. Introduction and Motivation

In this note we shall discuss and prove a theorem giving sufficient conditions for a factor to be either finite or purely infinite (type III). Since these conditions arise from physical considerations we shall use this introductory section to discuss the connection between these conditions and properties of the equilibrium states in quantum statistical mechanics. Some of the consequences of the theorem will be discussed in section 3.

Our starting point will be a C^*-algebra $\mathfrak{A}$ of quasi-local observables and a one parameter group of automorphisms $A \in \mathfrak{A} \to A_t \in \mathfrak{A}$ corresponding to time-evolution. We shall here take the point of view that a thermal equilibrium state ω is a positive linear form over the C^*-algebra $\mathfrak{A}$ satisfying the following conditions

1. ω is invariant, i. e., $\omega(A_t) = \omega(A)$.

2. $\omega(A^*A) = 0$ implies $A = 0$.

3. For fixed A and B $\omega(A_t B)$ is a function of t which can be continued analytically in the strip $0 > \operatorname{Im} t > \beta$ and is continuous on the boundaries. Similarly the function $\omega(BA_t)$ is analytical in the strip $0 < \operatorname{Im} t < \beta$, and

$$\omega(A_t B)_{t=t_0-i\beta} = \omega(BA_t)_{t=t_0}\,. \tag{1}$$