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Local Fields with the Wrong Connection Between Spin and Statistics *

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Abstract. It is proved that there exist free field operators which satisfy local commutativity and which transform according to certain unitary representations of the homogeneous Lorentz group. The fields satisfy axioms similar to the Wightman axioms, and give rise to local algebras of observables obeying postulates similar to those suggested by HAAG. They describe a tower of particles with spins $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}, \ldots$, but commute at space-like separation, giving rise to Bose statistics for the particles. This shows that the well-known theorem on spin and statistics cannot be extended to general theories of local observables; it also shows that the assumptions made in "S-matrix theory" do not hold for the S-matrix of a theory of interacting infinite fields.

1. Introduction

There has been some interest lately [1] in theories of elementary particles in which the particles fall into infinite sets which transform as a unitary representation of a non-compact symmetry group. This naturally led to the question of whether the theory can be described by a causal quantized field which transforms in the same way. The work of FELDMAN and MATTHEWS shows that for such fields the spin-statistics theorem of ordinary field theory is no longer valid; in particular, one can quantize a system of particles with spins $\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, \ldots$, with Bose statistics, and still describe them by a local field (in the more general sense).

On the other hand it has been conjectured that the spin-statistics theorem can be proved within the more general framework of the theory of localized observables [3, 4]. It is therefore worthwhile to give a proof of the existence of a concrete counter-example to this conjecture, by establishing some results similar to the work of [2] on a rigorous basis. This paper is devoted to this end.

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