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On Generalizations of Isoparity

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Abstract. The generalization of the isoparity (G-parity) for an arbitrary internal symmetry group is the problem of adjoining appropriately the charge conjugation operation, thereby extending the group. A complete solution to this problem is given, and explicitly detailed for the four families of compact, simple and simply-connected Lie groups (SU_n, $\overline{R}_{2k+1}$, $\overline{R}_{2k}$ and Sp_n).

The extended isoparity is shown to depend upon the structure of the groups in question, and the required structure is developed and summarized. The properties of the extended isoparity are discussed and two special cases — 'strong G-parity' and 'weak G-parity' — are treated in more detail.

I. Introduction and summary

The charge conjugation operation, $\mathscr{C}$, — more properly "matter-antimatter conjugation" — is a symmetry operation of quantum mechanics which briefly put, reverses the sign of all (generalized) charges for all states:

$$\mathscr{C}\,|A\,(Q_i)\rangle = \eta_A\,|A\,(-Q_i)\rangle$$

where η_A indicates the intrinsic charge conjugation parity of the state $|A\rangle$ and $|\eta_A|^2 = 1$.

We suppose moreover that there exists a continuous compact Lie group which acts on the states $|A\rangle$. For the special case where only ordinary charge and the isospin group are considered the operation of charge reversal, i.e., reflection in a plane containing the charge axis, can be combined in the well-known way [1] with a rotation (C) in isospace to yield a reflection operator reversing all directions in isospace; this is the usual one of the two possible isoparity operators (the G parity [2] (see also references in [2]) to use the customary but less descriptive term) defined as $G = \eta_A C\mathscr{C}$.

The present paper is concerned with the problem of generalizing the concept of the isoparity operator from the special example of the isospin group of internal symmetries to the larger class of internal symmetries

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