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On the Self-Adjointness of the Operator $-\Delta + V$

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Abstract. The essential self-adjointness of the operator $-\Delta + V$ is proved, where V is a potential whose main property is the high singularity and repulsiveness at the origin.

1. Introduction

The essential self-adjointness of the operator $-\Delta + V$ will be proved for a class of potentials V which are defined precisely through the conditions (1.1)—(1.5). Before writing these definitions, the qualitative description of the potential will be given briefly. The potential V is a real function $V(x, y, z)$ which is positive at the origin and its singularity there is higher than $1/r^2$ and independent of the way of approaching the origin. Outside the origin the potential $V(x, y, z)$ may possess singularities, such that the square of the potential is a locally integrable function. KATO [1] considered the same problem for another class of the potentials which essentially differ in the behaviour at the origin. The exact definition of the potential is:

The real function $V(x, y, z)$ can be decomposed in the form:

$$V(x, y, z) = V_1(x, y, z) + V_2(x, y, z) + P(r)\, Q(x, y, z)\ , \qquad (1.1)$$

where the four functions on the right of this decomposition satisfy the conditions:

$$\int V_1^2(x, y, z)\, r^{\varepsilon}\, dx\, dy\, dz < \infty, \quad \varepsilon > 0\ , \qquad (1.2)$$

$$\limsup_{x,y,z} |V_2(x, y, z)| < \infty\ , \qquad (1.3)$$

$$P(r) = \begin{cases} \frac{1}{r^{\alpha}}(\delta - r)^3 \quad \text{or} \quad e^{\frac{1}{r^{\beta}}}(\delta - r)^3\ , & r \leqq \delta\ , \\ 0 & r > \delta\ , \end{cases} \qquad (1.4)$$

where $\alpha > 2$ or $\beta > 0$,

$$\limsup_{K(d)} |Q(x, y, z) - Q_0| < q(d), \quad d < \delta\ , \qquad (1.5)$$

where $K(d)$ is the sphere of radius d centred at the origin, Q_0 is a positive constant and $q(d)$ is a monotonic continuous function which vanishes when d tends to zero. In the following we put $Q_0 = 1$ without loss of generality.

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