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Representations of O(N) Spin Models by Self-Avoiding Random Walks

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Abstract: We establish that correlation functions of classical lattice spin models can be represented by series expansions in terms of self-avoiding random walks. Using this, we get new upper bounds of critical temperatures of the $O(N)$ symmetric classical Heisenberg models.

1. Introduction

Based on the idea of Symanzik [20], the authors of [5, 4, 9] formulated the random walk representations of classical lattice spin systems and used them to derive various correlation inequalities and bounds for the critical inverse temperatures β_c. We tried to combine the idea of renormalization group with the random walk representations, and succeeded in the first step of transformations of block spin type. Namely we could renormalize the contribution of the smallest loops (self-crossing points) in the expansion as the changes of the single spin distributions and obtain an improvement of β_c for the $O(N)$ Heisenberg model [10, 11], in which the method of blockwise diagonalization of matrices is used to remove smallest loops from the random walk.

The purpose of this paper is to show that all loops can be removed from the random walk representations. In other words, we give a self-avoiding random walk representation of correlation functions of classical lattice spin systems, by which we obtain a new lower bound of β_c of the $O(N)$ Heisenberg model. It is better than the bound in [11] and is the most accurate among the theoretical values so far obtained. See the table. For example, we recover $\beta_c = \infty$ for every N on the one dimensional lattice, and we expect that this provides us with new methods to solve the long standing conjecture of non-existence of

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