Commun. Math. Phys. 183, 661–705 (1997)

Communications in
Mathematical
Physics

Yang-Mills on Surfaces with Boundary: Quantum Theory and Symplectic Limit

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Received: 30 June 1996 / Accepted: 22 July 1996

Abstract: The quantum field measure for gauge fields over a compact surface with boundary, with holonomy around the boundary components specified, is constructed. Loop expectation values for general loop configurations are computed. For a compact oriented surface with one boundary component, let $\mathcal{M}(\Theta)$ be the moduli space of flat connections with boundary holonomy lying in a conjugacy class Θ in the gauge group G. We prove that a certain natural closed 2−form on $\mathcal{M}(\Theta)$, introduced in an earlier work by C. King and the author, is a symplectic structure on the generic stratum of $\mathcal{M}(\Theta)$ for generic Θ. We then prove that the quantum Yang-Mills measure, with the boundary holonomy constrained to lie in Θ, converges in a natural sense to the corresponding symplectic volume measure in the classical limit. We conclude with a detailed treatment of the case $G = SU(2)$, and determine the symplectic volume of this moduli space.

1. Introduction and Overview of Results

This paper presents the construction of a quantum gauge field measure over compact surfaces, with specified boundary holonomies, and a determination of the classical limit of this measure when the surface is oriented and has one boundary component.

Results concerning the quantum field measure. The construction of the measure and determination and study of the loop expectation values are carried out in Sects. 1–5. In these sections:

(i) We construct the Euclidean quantum field measure for gauge theory over a compact surface with boundary, with boundary holonomy (or its conjugacy class) specified (the gauge group is a compact connected Lie group).

(ii) Loop expectation values are computed explicitly, and it is shown that they are invariant under appropriate area-preserving surface homeomorphisms.