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Quantum Field Theory on Spacetimes with a Compactly Generated Cauchy Horizon

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Abstract. We prove two theorems which concern difficulties in the formulation of the quantum theory of a linear scalar field on a spacetime, (M, g_{ab}), with a compactly generated Cauchy horizon. These theorems demonstrate the breakdown of the theory at certain *base points* of the Cauchy horizon, which are defined as 'past terminal accumulation points' of the horizon generators. Thus, the theorems may be interpreted as giving support to Hawking's 'Chronology Protection Conjecture', according to which the laws of physics prevent one from manufacturing a 'time machine'. Specifically, we prove:

Theorem 1. *There is no extension to (M, g_{ab}) of the usual field algebra on the initial globally hyperbolic region which satisfies the condition of F-locality at any base point. In other words, any extension of the field algebra must, in any globally hyperbolic neighbourhood of any base point, differ from the algebra one would define on that neighbourhood according to the rules for globally hyperbolic spacetimes.*

Theorem 2. *The two-point distribution for any Hadamard state defined on the initial globally hyperbolic region must (when extended to a distributional bisolution of the covariant Klein-Gordon equation on the full spacetime) be singular at every base point x in the sense that the difference between this two point distribution and a local Hadamard distribution cannot be given by a bounded function in any neighbourhood (in $M \times M$) of (x, x).*

In consequence of Theorem 2, quantities such as the renormalized expectation value of ϕ^2 or of the stress-energy tensor are necessarily ill-defined or singular at any base point.

The proof of these theorems relies on the 'Propagation of Singularities' theorems of Duistermaat and Hörmander.

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