Commun. Math. Phys. 183, 423–461 (1997)

Communications in Mathematical Physics

Extensions of the Matrix Gelfand–Dickey Hierarchy from Generalized Drinfeld–Sokolov Reduction

László Fehér[1,★], Ian Marshall[2]

[1] Department of Physics, University of Wales Swansea, Singleton Park, Swansea SA2 8PP, U.K.
[2] Department of Mathematics, Leeds University, Leeds LS2 9JT, U.K.
E-mail: amt6im@amsta.leeds.ac.uk

Received: 10 April 1995 / Accepted: 11 July 1996

Abstract: The $p \times p$ matrix version of the r-KdV hierarchy has been recently treated as the reduced system arising in a Drinfeld–Sokolov type Hamiltonian symmetry reduction applied to a Poisson submanifold in the dual of the Lie algebra $\widehat{gl}_{pr} \otimes \mathbb{C}[\lambda, \lambda^{-1}]$. Here a series of extensions of this matrix Gelfand–Dickey system is derived by means of a generalized Drinfeld–Sokolov reduction defined for the Lie algebra $\widehat{gl}_{pr+s} \otimes \mathbb{C}[\lambda, \lambda^{-1}]$ using the natural embedding $gl_{pr} \subset gl_{pr+s}$ for s any positive integer. The hierarchies obtained admit a description in terms of a $p \times p$ matrix pseudo-differential operator comprising an r-KdV type positive part and a non-trivial negative part. This system has been investigated previously in the $p = 1$ case as a constrained KP system. In this paper the previous results are considerably extended and a systematic study is presented on the basis of the Drinfeld–Sokolov approach that has the advantage that it leads to local Poisson brackets and makes clear the conformal ($\mathcal{W}$-algebra) structures related to the KdV type hierarchies.

0. Introduction

This paper is a continuation of [1], where it was shown how the matrix Gelfand–Dickey hierarchy [2, 3] fits into the Drinfeld–Sokolov approach [4] (see also [5–9]) to generalized KdV hierarchies.

The phase space of the matrix Gelfand–Dickey hierarchy is the space of $p \times p$ matrix Lax operators

$$L_{p,r} = P\partial^r + u_1\partial^{r-1} + \cdots + u_{r-1}\partial + u_r, \quad u_i \in C^\infty(S^1, gl_p), \tag{0.1}$$

where P is a diagonal constant matrix with distinct, non-zero entries. This phase space has two compatible Poisson brackets: the linear and quadratic matrix Gelfand–Dickey Poisson brackets. The Hamiltonians generating a commuting hierarchy of bihamiltonian flows are obtained from the residues of the componentwise fractional

★ On leave from Theoretical Physics Department of Szeged University, H-6720 Szeged, Hungary