Commun. Math. Phys. 183, 183–203 (1997)

Communications in Mathematical Physics

Symmetric Instantons and the ADHM Construction

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Received: 13 June 1994/Accepted: 27 June 1996

Abstract: We construct all $SU(2)$ Yang–Mills instantons on S^4 that admit a certain symmetry ("quadrupole symmetry"). This is accomplished by an equivariant version of the "ADHM monad" classification of instantons. This work is part of an attempt to better understand the structure of non-self-dual Yang–Mills connections with the same symmetry.

1. Introduction

A. Statement of Results. An *instanton,* in this paper, refers to a unitary connection with anti-self-dual curvature on a rank-two hermitian vector bundle over the standard four-sphere S^4. Such a bundle is determined, up to an isomorphism, by its second Chern number c_2, and admits instantons if and only if $c_2 \geqq 0$. For a detailed account of the theory of instantons on S^4, see for example the book [5].

This article is devoted to the study of instantons with "quadrupole symmetry" [4]. To define these, let the orthogonal group $SO(3)$ act on $S^4 \subset \mathbb{R}^5$ via its irreducible linear representation on $\mathbb{R}^5$ (conjugation of traceless symmetric 3×3 real "quadrupole" matrices). Then a *bundle with quadrupole symmetry* consists of a rank-two hermitian vector bundle over S^4 together with a lift of the $SO(3)$-action on S^4 to a unitary action on the bundle. In general, to construct such lifts, one needs to pass from $SO(3)$ to its double-cover $\mathrm{Spin}(3) \cong SU(2)$. Finally, an *instanton with quadrupole symmetry,* or simply a *symmetric instanton,* consists of a bundle with quadrupole symmetry together with an instanton connection which is invariant under the $SU(2)$-action on the bundle.

The classification of bundles with quadrupole symmetry is quite simple, given by a pair of odd positive integers (n_+, n_-). The significance of these integers is the following: the singular locus of the $SU(2)$-action on S^4 consists of exactly two orbits; for a point on one of these orbits the identity component of the stabilizer subgroup is a circle group which acts on the fiber with weights $\{n_+, -n_+\}$ or

★ J.S. was supported by NSF Grants DMS-9106807 and DMS-9404468
Part of this work was done at the 1991 Regional Geometry Institute in Park City, Utah