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A System of Difference Equations With Elliptic Coefficients and Bethe Vectors

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Abstract: An elliptic analogue of the q deformed Knizhnik–Zamolodchikov equations is introduced. A solution is given in the form of a Jackson-type integral of Bethe vectors of the XYZ-type spin chains.

Introduction

In this paper we introduce a holonomic system of difference equations associated to elliptic R matrices and give its solution in the form of a Jackson-type integral, following Reshetikhin's idea [R] for the trigonometric R matrices.

Reshetikhin constructed a solution to the q-deformed Knizhnik–Zamolodchikov equations [FR] by a Jackson-type integration of Bethe vectors of the XXZ-type spin chain models. Matsuo [Ma] also found the same kind of formulae from a different viewpoint.

On the other hand, the Bethe Ansatz method for the spin chain models associated to the elliptic R matrices has been studied since Baxter [B]. Hence a natural question is how to find an elliptic version of Reshetikhin's approach to the q-KZ equation. It turns out that the argument in [R] can be carried out for the elliptic R matrices as well, except for one point. In contrast to the trigonometric case, an elliptic spin chain model does not have a unique vacuum vector in its local state space but a series of "pseudo-vacua" which depend non-trivially on a spectral parameter. This dependence breaks down naive analogy.

We overcome this difficulty by introducing a "space of Bethe vectors" and a boundary operator which shifts a spectral parameter.

In the vertex picture the linear space of Bethe vectors depends on spectral parameters. Therefore we have to use the IRF picture in order to interpret the system as a holonomic matrix difference system in the sense of Aomoto [A]. In this context, our system is described in terms of representation of Felder's elliptic quantum groups [F].

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