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The Behavior of the Weyl Function in the Zero-Dispersion KdV Limit

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Dedicated to Peter Lax on his 70^{th} birthday

Abstract: The moment formulas that globally characterize the zero-dispersion limit of the Korteweg–deVries (KdV) equation are known to be expressed in terms of the solution of a maximization problem. Here we establish a direct relation between this maximizer and the zero-dispersion limit of the logarithm of the Jost functions associated with the inverse spectral transform. All the KdV conserved densities are encoded in the spatial derivative of these functions, known as Weyl functions. We show the Weyl functions are densities of measures that converge in the weak sense to a limiting measure. This limiting measure encodes all of the weak limits of the KdV conserved densities. Moreover, we establish the weak limit of spectral measures associated with the Dirichlet problem.

1. Introduction

This paper presents a global interpretation of the moment formulas that characterize the zero-dispersion limit of the Korteweg–deVries equation (KdV) and which were first described in [18, 19]. The problem is to determine the limit

$$\lim_{\varepsilon \to 0} u^{\varepsilon}(x,t)\,, \tag{1.1}$$

where u^{ε} solves the initial-value problem

$$\partial_t u^{\varepsilon} - 6u^{\varepsilon}\partial_x u^{\varepsilon} + \varepsilon^2 \partial_{xxx} u^{\varepsilon} = 0\,, \tag{1.2a}$$

$$u^{\varepsilon}(x,0) = v(x)\,, \tag{1.2b}$$

for v of scattering class and independent of ε. More generally, the limit (1.1) can be considered for $u^{\varepsilon}(x,\mathbf{t})$, the simultaneous solution of the whole KdV hierarchy of commuting flows with initial data given by (1.2b). Here $\mathbf{t} = (t_0, t_1 \cdots)$ denotes an infinite vector of times corresponding to the KdV flows where all but finitely many of the t_m are zero.

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