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On Spectral Asymptotics for Domains with Fractal Boundaries

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Abstract: We discuss the spectral properties of the Laplacian for domains Ω with fractal boundaries. The main goal of the article is to find the second term of spectral asymptotics of the counting function $N(\lambda)$ or its integral transformations: Θ-function, ζ-function. For domains with smooth boundaries the order of the second term of $N(\lambda)$ (under "billiard condition") is one half of the dimension of the boundary. In the case of fractal boundaries the well-known Weyl–Berry hypothesis identifies it with one half of the Hausdorff dimension of $\partial\Omega$, and the modified Weyl–Berry conjecture with one half of the Minkowski dimension of $\partial\Omega$. We find the spectral asymptotics for three natural broad classes of fractal boundaries (cabbage type, bubble type and web type) and show that the Minkowski dimension gives the proper answer for cabbage type of boundaries (due to "one dimensional structure" of the cabbage type fractals), but the answers are principally different in the two other cases.

Contents

1. Introduction

The classical Weyl–Berry conjecture is related to the spectral counting function $N(\lambda)$ for the Laplacian in a bounded domain $\Omega \subset \Re^d$, $d \geqq 1$, with smooth boundary $\partial\Omega$. Let us consider the spectral problems for the Dirichlet Laplacian $-\Delta^-$:

$$-\Delta\Psi = \lambda\Psi \quad \text{on } \Omega, \qquad \Psi = 0 \quad \text{on } \partial\Omega$$