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# Deformation Quantization and Nambu Mechanics

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**Abstract:** Starting from deformation quantization (star-products), the quantization problem of Nambu Mechanics is investigated. After considering some impossibilities and pushing some analogies with field quantization, a solution to the quantization problem is presented in the novel approach of Zariski quantization of fields (observables, functions, in this case polynomials). This quantization is based on the factorization over $\mathbb{R}$ of polynomials in several real variables. We quantize the infinite-dimensional algebra of fields generated by the polynomials by defining a deformation of this algebra which is Abelian, associative and distributive. This procedure is then adapted to derivatives (needed for the Nambu brackets), which ensures the validity of the Fundamental Identity of Nambu Mechanics also at the quantum level. Our construction is in fact more general than the particular case considered here: it can be utilized for quite general defining identities and for much more general star-products.

## 1. Introduction

*1.1 Nambu Mechanics.* Nambu proposed his generalization of Hamiltonian Mechanics [17] by having in mind a generalization of the Hamilton equations of motion which allows the formulation of a statistical mechanics on $\mathbb{R}^3$. He stressed that the only feature of Hamiltonian Mechanics that one needs to retain for that purpose, is the validity of the Liouville theorem. In that spirit, he considered the following equation of motion:

$$\frac{d\boldsymbol{r}}{dt} = \nabla g(\boldsymbol{r}) \wedge \nabla h(\boldsymbol{r}) \ , \quad \boldsymbol{r} = (x, y, z) \in \mathbb{R}^3, \tag{1}$$

where $x$, $y$, $z$ are the dynamical variables and $g$, $h$ are two functions of $\boldsymbol{r}$. Then the Liouville theorem follows directly from the identity:

$$\nabla \cdot (\nabla g(\boldsymbol{r}) \wedge \nabla h(\boldsymbol{r})) = 0 \ ,$$

which tells us that the velocity field in Eq. (1) is divergenceless.

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