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Ground States of Fermions on Lattices

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Abstract: We consider Fermion systems on integer lattices. We establish the existence of dynamics for a class of long range interactions. The infinite volume ground states are considered. The equivalence of the variational principle and ground state conditions is proved for long range interactions. We also prove that any pure translationally invariant ground state of the gauge invariant algebra is extendible to a ground state of the full CAR algebra for the Hamiltonian with a chemical potential (equivalence of ensemble for canonical and ground canonical states at the zero temperature).

1. Introduction

In this paper we consider lattice Fermion Hamiltonians and their ground states. We consider the infinite volume systems directly by use of functional analytic techniques. The main object of this paper is as follows. (1) We establish the existence of the time evolution as the one-parameter group of automorphisms on the algebra of observables (Heisenberg picture). (2) We show that the infinite volume ground state in the sense of (1.13) below is characterized via the minimization of energy. Equation (1.13) means the positive energy representation while the principle of minimization of energy expectation value is the Gibbs variational principle at zero temperature. (3) Any translationally invariant pure ground state of the gauge invariant algebra can be extended to a ground state of the full CAR algebra for the Hamiltonian with a chemical potential term.

The prototype of the interaction we have in mind is the spinless *translationally invariant* Hamiltonian H,

$$H = \sum_{\mathbf{Z}^d \ni k,l} t_{kl} a_k^* a_l + \sum_{A} W_A \prod_{A \ni j} n_j \,, \tag{1.1}$$

where $a_k^* a_l$ and n_j are Fermion creation, annihilation and number operators and W_A is a real number (=coupling constant) depending on the finite subset A of $\mathbf{Z}^d$. The translational invariance means that $t_{ij} = t_{0\,i-j}$ and that $W_A = W_{A-j}$. The decay