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A Low Temperature Expansion for Classical N-Vector Models. II. Renormalization Group Equations

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Abstract: This paper continues the analysis of the low temperature expansions for classical N-vector models started in [1]. A main part of it is a derivation of renormalization group equations and a construction of their solutions. To do this we have to introduce "a fluctuation integral" connected with a next renormalization transformation, and to make its preliminary analysis. The results of the paper are summarized in theorems stating that the renormalization transformation preserves the space of densitites, or actions described inductively in [1].

1. Introduction, Formulation of Main Theorems

This paper is a continuation of the paper [1], and we use the notations, definitions and results established there, as well as in the following papers [2, 3]. Our main purpose here is to define renormalization procedures for new contributions to the effective actions and the generating functionals, and to set up and analyse renormalization group equations for the "running" coefficients in the main term of the effective action. The new contributions come from "small field" fluctuation integrals connected with a successive $k + 1^{st}$ renormalization transformation, and our other purpose is to set up such fluctuation integrals, and to define the new contributions. The renormalization procedures, and in particular the renormalization group equations have to be defined under the assumption that the effective actions are defined on the whole lattice, i.e., in the framework of paper [1]. To give some more precise shape to this paper we assume also that the fluctuation integrals are defined on the whole lattice, i.e., the lattice is "a small field region," but it is very easy to generalize constructions and results obtained in this case to general cases when small field regions are subregions of the whole lattice. If the renormalization procedure and the renormalization group flow depended on such a region, then a new renormalized

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