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Strong Connections on Quantum Principal Bundles

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Abstract: A gauge invariant notion of a strong connection is presented and characterized. It is then used to justify the way in which a global curvature form is defined. Strong connections are interpreted as those that are induced from the base space of a quantum bundle. Examples of both strong and non-strong connections are provided. In particular, such connections are constructed on a quantum deformation of the two-sphere fibration $S^2 \to RP^2$. A certain class of strong $U_q(2)$-connections on a trivial quantum principal bundle is shown to be equivalent to the class of connections on a free module that are compatible with the q-dependent hermitian metric. A particular form of the Yang–Mills action on a trivial $U_q(2)$-bundle is investigated. It is proved to coincide with the Yang–Mills action constructed by A. Connes and M. Rieffel. Furthermore, it is shown that the moduli space of critical points of this action functional is independent of q.

Introduction

Two of the mainstreams of Noncommutative Geometry concentrate around the notions of a projective module [12, 14] and of a quantum group [24, 38]. Quite recently (see [9, 22, 28]), the concept of a quantum principal bundle was systematically developed with quantum groups (Hopf algebras) in the role of structure groups. Hence, since both projective modules and quantum principal bundles serve as starting points for quantum geometric considerations, the conceptual framework provided by the notion of a quantum principal bundle has a good chance of unifying those two branches of Noncommutative Geometry.

In the classical differential geometry, it is hard to overestimate the interplay between Lie groups and K-theory. Therefore, it is natural to expect that establishing a similar interaction in the noncommutative case is necessary for better understanding of quantum geometry. It is already known that the classification of quantum

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