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GKZ-Generalized Hypergeometric Systems in Mirror Symmetry of Calabi–Yau Hypersurfaces

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Abstract: We present a detailed study of the generalized hypergeometric system introduced by Gel'fand, Kapranov and Zelevinski (GKZ-hypergeometric system) in the context of toric geometry. GKZ systems arise naturally in the moduli theory of Calabi–Yau toric varieties, and play an important role in applications of the mirror symmetry. We find that the Gröbner basis for the so-called toric ideal determines a finite set of differential operators for the local solutions of the GKZ system. At the special point called the large radius limit, we find a close relationship between the principal parts of the operators in the GKZ system and the intersection ring of a toric variety. As applications, we analyze general three dimensional hypersurfaces of Fermat and non-Fermat types with Hodge numbers up to $h^{1,1} = 3$. We also find and analyze several non-Landau–Ginzburg models which are related to singular models.

1. Introduction

Recent studies on nonperturbative aspects of string theory have made remarkable progress in understanding the structure of moduli spaces in string theory. Applications of mirror symmetry, for example, in type II string compactification to studying the geometry of moduli spaces is one of the most successful developments. Starting from the pioneering work by Candelas et al. [1], and subsequently by others, the quantum geometry of the moduli spaces for many Calabi–Yau models [2–11] have now been well understood via mirror symmetry. At the same time, there is parallel progress in studying the axiomatic framework of quantum geometry and its application to enumerative geometry [12]. Also in explicit constructions of the geometry of concrete Calabi–Yau models, it is now understood that for a large class of Calabi–Yau varieties, the mirror maps have remarkable modular and integrality properties [13–15]. These models present strong and even beautiful evidence for the recent proposal for the so-called type II-heterotic string duality [16]. These Calabi–Yau models continue to provide fruitful testing ground for string duality [17].