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Schur Duality in the Toroidal Setting

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Abstract: The classical Frobenius–Schur duality gives a correspondence between finite dimensional representations of the symmetric and the linear groups. The goal of the present paper is to extend this construction to the quantum toroidal setup with only elementary (algebraic) methods. This work can be seen as a continuation of [J, D1 and C2] (see also [C-P and G-R-V]) where the cases of the quantum groups $\mathbf{U}_q(\mathfrak{sl}(n))$, $\mathbf{Y}(\mathfrak{sl}(n))$ (the Yangian) and $\mathbf{U}_q(\hat{\mathfrak{sl}}(n))$ are given. In the toroidal setting the two algebras involved are deformations of Cherednik's double affine Hecke algebra introduced in [C1] and of the quantum toroidal group as given in [G-K-V]. Indeed, one should keep in mind the geometrical construction in [G-R-V] and [G-K-V] in terms of equivariant K-theory of some flag manifolds. A similar K-theoretic construction of Cherednik's algebra has motivated the present work. At last, we would like to lay emphasis on the fact that, contrary to [J, D1 and C2], the representations involved in our duality are infinite dimensional. Of course, in the classical case, i.e., $q = 1$, a similar duality holds between the toroidal Lie algebra and the toroidal version of the symmetric group.

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1. Definition of the Toroidal Hecke Algebra

For any positive integer k set $[k] = \{0, 1, 2, \ldots, k\}$ and $[k]^\times = \{1, 2, \ldots, k\}$.

1.1. Definition. *The toroidal Hecke algebra of type* $\mathfrak{gl}(l)$, $\ddot{\mathbf{H}}_{\mathscr{A}}$, *is the unital associative algebra over* $\mathscr{A} = \mathbb{C}[\mathbf{x}^{\pm 1}, \mathbf{y}^{\pm 1}, \mathbf{q}^{\pm 1}]$ *with generators*

$$\mathrm{T}_i^{\pm 1}, \mathrm{X}_j^{\pm 1}, \mathrm{Y}_j^{\pm 1}, \quad i \in [l-1]^\times,\ j \in [l]^\times\ ,$$

and the following relations:

$$\mathrm{T}_i\mathrm{T}_i^{-1} = \mathrm{T}_i^{-1}\mathrm{T}_i = 1, \qquad (\mathrm{T}_i + 1)(\mathrm{T}_i - \mathbf{q}^2) = 0\ ,$$

$$\mathrm{T}_i\mathrm{T}_{i+1}\mathrm{T}_i = \mathrm{T}_{i+1}\mathrm{T}_i\mathrm{T}_{i+1}\ ,$$