Commun. Math. Phys. 182, 457 – 468 (1996)

Communications in Mathematical Physics

The Characteristic Exponents of the Falling Ball Model

Nándor Simányi★

Department of Mathematics, The Pennsylvania State University, University Park, PA 16802, USA

Received: 5 April 1996 / Accepted: 15 May 1996

Abstract: We study the characteristic exponents of the Hamiltonian system of n ($\geqq 2$) point masses $m_1, \dots, m_n$ freely falling in the vertical half line $\{q \mid q \geqq 0\}$ under constant gravitation and colliding with each other and the solid floor $q = 0$ elastically. This model was introduced and first studied by M. Wojtkowski. Hereby we prove his conjecture: All relevant characteristic (Lyapunov) exponents of the above dynamical system are nonzero, provided that $m_1 \geqq \cdots \geqq m_n$ (i.e. the masses do not increase as we go up) and $m_1 \neq m_2$.

1. Introduction

In his paper [W-I] M. Wojtkowski introduced the following Hamiltonian dynamical system with discontinuities: There is a vertical half line $\{q \mid q \geqq 0\}$ given and n ($\geqq 2$) point particles with masses $m_1 \geqq m_2 \geqq \cdots \geqq m_n > 0$ and positions $0 \leqq q_1 \leqq q_2 \leqq \cdots \leqq q_n$ are moving on this half line so that they are subjected to a constant gravitational acceleration $a = -1$ (they fall down), they collide elastically with each other, and the first (lowest) particle also collides elastically with the hard floor $q = 0$. We fix the total energy

$$H = \sum_{i=1}^{n} \left(m_i q_i + \frac{1}{2} m_i \dot{q}_i^2 \right)$$

by taking $H = 1$. The arising Hamiltonian flow with collisions $(\mathbf{M}, \{\psi^t \mid t \in \mathbb{R}\}, \mu)$ (μ is the Liouville measure) is the subject of this paper.

Before formulating the result of this article, however, it is worth mentioning here three important facts:

(1) Since the phase space $\mathbf{M}$ is compact, the Liouville measure μ is finite.

(2) The phase points $x \in \mathbf{M}$ for which the orbit $\{\psi^t(x) \mid t \in \mathbb{R}\}$ hits at least one singularity (i.e. a multiple collision) are contained in a countable union of

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★On leave from the Mathematical Institute of the Hungarian Academy of Sciences, H-1364, Budapest, P.O.B. 127, Hungary. Research partially supported by the Hungarian National Foundation for Scientific Research, No. 16425. E-mail: simanyi@math.psu.edu