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Heat-Kernels and Functional Determinants on the Generalized Cone

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Abstract: We consider zeta functions and heat-kernel expansions on the bounded, generalized cone in arbitrary dimensions using an improved calculational technique. The specific case of a global monopole is analysed in detail and some restrictions thereby placed on the $A_{5/2}$ coefficient. The computation of functional determinants is also addressed. General formulas are given and known results are incidentally, and rapidly, reproduced.

1. Introduction

In this paper we refine and generalise techniques developed earlier for the evaluation of heat-kernel expansion coefficients and functional determinants of elliptic operators on manifolds with boundary. We concentrate on ball-like manifolds because precise answers can be found and, apart from illustrating our method, the results for such specific manifolds are often useful in restricting the general forms of heat-kernel coefficients.

One of the motivations for this paper is to compute for a particular curved manifold whose boundary is not geodesically embedded. The resulting restrictions are a little more informative than some others available [28–30]. The manifold also possesses a singularity, which increases its interest.

For calculational simplicity the operator is taken to be the modified Laplacian, $\Delta - \xi R$, acting on scalars. The analysis could be extended to forms without difficulty and also to other fields with a certain amount of extra work [23, 5, 24–26, 38, 31]. It is possible that our techniques will be of value in areas of physics where finite size systems and boundary effects play a role, such as quantum cosmology and statistical mechanics.

In the next section we outline the geometry we have in mind and discuss the eigenmodes. The ζ-function is next constructed in Sect. 3 and its properties translated into heat-kernel language in the following section. In order to make this paper reasonably self-contained the techniques alluded to previously are restated in improved and compactified form. The general method is applied to a global monopole in Sect. 5 and the results used in Sect. 7 to place restrictions on the numerical