Commun. Math. Phys. 182, 355–370 (1996)

Communications in Mathematical Physics

On Lieb–Thirring Inequalities for Higher Order Operators with Critical and Subcritical Powers

Y. Netrusov[1,3], **T. Weidl**[1,2]

[1] School of Mathematical Sciences, University of Sussex at Brighton, BN1 9QH Brighton Falmers, UK
[2] Royal Institute of Technology, Department of Mathematics, S-10044 Stockholm, Sweden
[3] POMI, Fontanka 27, 191011 St. Petersburg, Russia

Received: 19 April 1996 / Accepted: 29 April 1996

Abstract: Let $\varkappa_i(H_l(V))$ denote the negative eigenvalues of the operator $H_l(V)u := (-\Delta)^l u - V(x)u,\ V \geqq 0,\ x \in \mathbb{R}^d$ on $L_2(\mathbb{R}^d)$. We prove the two-sided estimate

$$\tilde{\mathfrak{L}}(d,l)\int_{\mathbb{R}^d} V(x)dx \leqq \sum_k |\varkappa_k(H_l(V))|^{1-\kappa} \leqq \mathfrak{L}(d,l,1-\kappa)\int_{\mathbb{R}^d} V(x)dx, \quad \kappa = d/2l < 1\ .$$

We discuss bounds on the Riesz means $\sum_k |\varkappa_k(H_l(V))|^\mu$ if $0 < \mu < 1 - \kappa$.

1. Introduction

1.1. We consider the quadratic form

$$\mathbf{h}_l(V)[u,u] := \int_{\mathbb{R}^d} |\nabla^l u|^2 dx - \int_{\mathbb{R}^d} V|u|^2 dx, \quad 0 \leqq V \in L_1^{\mathrm{loc}}(\mathbb{R}^d), \quad l \in \mathbb{N}_+\ ,$$

defined on functions $u \in C_0^\infty(\mathbb{R}^d)$. If the function V vanishes properly at infinity, this form can be closed. Its closure generates the self-adjoint operator

$$H_l(V) := (-\Delta)^l - V(x) \tag{1}$$

on $L_2(\mathbb{R}^d)$, the negative spectrum of which is discrete and bounded from below. Let $\{\varkappa_k(H_l(V))\}$ stand for the non-decreasing, finite or infinite sequence of the negative eigenvalues of the operator $H_l(V)$.

Estimates on the negative spectrum of operators $H_l(V)$ in terms of the potential V have been studied for many years, see e.g. [3, 6, 17, 16, 8, 14, 13, 9]. For given d, l define

$$\kappa = \kappa(d,l) := \frac{d}{2l}, \qquad \nu = \nu(d,l) := 1 - \frac{d}{2l}\ . \tag{2}$$

The first author was supported by the EPSRC grant GR/J 32084. The second author was supported by *Deutsche Forschungsgemeinschaft* grant We 1964-1.