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Quantization of Solitons and the Restricted Sine-Gordon Model

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Abstract: We show how to compute form factors, matrix elements of local fields, in the restricted sine-Gordon model, at the reflectionless points, by quantizing solitons. We introduce (quantum) separated variables in which the Hamiltonians are expressed in terms of (quantum) τ-functions. We explicitly describe the soliton wave functions, and we explain how the restriction is related to an unusual hermitian structure. We also present a semi-classical analysis which enlightens the fact that the restricted sine-Gordon model corresponds to an analytical continuation of the sine-Gordon model, intermediate between sine-Gordon and KdV.

1. Introduction

About 20 years ago the work on quantization of integrable models of Quantum Field Theory started with the idea of quantizing the classical soliton solutions [1, 2]. Important results were achieved in this way, in particular for the sine-Gordon (SG) theory, the semi-classical spectrum of excitations (which happens to be exact quantum-mechanically) and the semi-classical approximation for the soliton S-matrix were found. The semi-classical S-matrix allowed to guess the exact S-matrix in the reflectionless case [2], and this was used further as a fundamental input in the bootstrap construction of the S-matrix for arbitrary coupling [3]. Later, however the idea of direct quantization of solitons was abandoned in favor of other approaches such as Bethe Ansatz and its algebraic formulation in the Quantum Inverse Scattering Method (QISM).

Whatever the original motivations and methods were, it is fair to say that the most significant results in the theory were obtained by bootstrap methods. Exact S-matrices [3, 4] and exact form factors [5, 6] were found by this method. Since

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