Supersymmetric σ-Models, Twistors, and the Atiyah–Hitchin Metric

Ivan T. Ivanov[1], M. Roček[2]

Institute for Theoretical Physics, State University of New York, Stony Brook, NY 11794-3840, USA

Abstract: The Legendre transform and its generalizations, originally found in supersymmetric σ-models, are techniques that can be used to give *local* constructions of hyperkähler metrics. We give a twistor space interpretation to the generalizations of the Legendre transform construction. The Atiyah–Hitchin metric on the moduli space of two monopoles is used as a detailed example.

1. Introduction

Some time ago, we discoverd several constructions of hyperkähler metrics by studying extended supersymmetric nonlinear sigma-models [1, 2, 3, 4]. In the hyperkähler quotient framework, the topology of the manifold and the completeness of the metric are accessible. On the other hand, the Legendre transform construction and its generalizations are purely local. They can be considered as changes of variables that linearize the Monge–Ampère equations governing hyperkähler metrics. In some cases, the Legendre transform construction *has* been used to find complete metrics. A twistor space interpretation has also been worked out. In this paper, we consider the simplest generalization of the Legendre transform construction [4]. We reformulate it using twistors and emphasize the special properties of the twistor spaces used. Throughout this paper we consider only the four dimensional case to avoid congesting the notation, but all the techniques are easily generalized to higher dimensions. As the main example we show that the hyperkähler metric in the moduli space of two monopoles [7] can be obtained by a generalized Legendre transform construction from a single simple function. Using this example we also propose various ingredients in the construction of new complete metrics.

In Section two we introduce the generalized Legendre transform construction from [4]. Section three contains a brief and, we hope, readable outline of the basics of twistor space. In Section four, the twistor interpretation of the generalized Legendre transform construction is worked out. A careful treatment of the singularities

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[1] E-mail: iti@insti.physics.sunysb.edu.
[2] E-mail: rocek@insti.physics.sunysb.edu.