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Coherent States for Quantum Compact Groups

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Dedicated to Professor L.D. Faddeev on his 60th birthday

Abstract: Coherent states are introduced and their properties are discussed for simple quantum compact groups A_l, B_l, C_l and D_l. The multiplicative form of the canonical element for the quantum double is used to introduce the holomorphic coordinates on a general quantum dressing orbit. The coherent state is interpreted as a holomorphic function on this orbit with values in the carrier Hilbert space of an irreducible representation of the corresponding quantized enveloping algebra. Using Gauss decomposition, the commutation relations for the holomorphic coordinates on the dressing orbit are derived explicitly and given in a compact R-matrix formulation (generalizing this way the q-deformed Grassmann and flag manifolds). The antiholomorphic realization of the irreducible representations of a compact quantum group (the analogue of the Borel–Weil construction) is described using the concept of coherent state. The relation between representation theory and non-commutative differential geometry is suggested.

1. Introduction

It is difficult to overestimate the importance of the concept of coherent states in theoretical and mathematical physics. They found various applications in quantum optics, quantum field theory, quantum statistical mechanics and other branches of physics as well as in some purely mathematical problems [21, 34]. The last-named include Lie group representations, special functions, automorphic functions, reproducing kernels, etc. In the Lie group representation theory there is a remarkable relation between the geometry on the coadjoint orbits and the irreducible representations, which is reflected by the method of orbits (geometric quantization) due to Kirillov, Kostant and Souriau [53]. On the other hand the concept of coherent states leads naturally to Berezin's quantization scheme [5]. The important sources of both methods are induced representations and the Borel–Weil theory. The intrinsic relationship between the geometric and Berezin quantization has been established. There are many papers devoted to this subject (e.g. [32, 37] and many others). Recently the coherent states were used to construct examples of non-commutative manifolds [14].