Topological Landau–Ginzburg Theory with a Rational Potential and the Dispersionless KP Hierarchy

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Abstract: Based on the dispersionless KP (dKP) theory, we study a topological Landau–Ginzburg (LG) theory characterized by a rational potential. Writing the dKP hierarchy in a general form treating all the primaries in an equal basis, we find that the hierarchy naturally includes the dispersionless (continuous) limit of Toda hierarchy and its generalizations having a finite number of primaries. Several flat solutions of the topological LG theory are obtained in this formulation, and are identified with those discussed by Dubrovin. We explicitly construct gravitational descendants for all the primary fields. Giving a residue formula for the 3-point functions of the fields, we show that these 3-point functions satisfy the topological recursion relation. The string equation is obtained as the generalized hodograph solutions of the dKP hierarchy, which show that all the gravitational effects to the constitutive equations (2-point functions) can be renormalized into the coupling constants in the small phase space.

1. Introduction

A coupled system of the gravity and topological matter fields ϕ^α (primaries) is given by the action

$$S = S_0 + \sum_{\substack{\alpha \in \text{primaries} \\ N \geq 0}} t_{N,\alpha} \int_\Sigma \sigma_N(\phi^\alpha) \,, \qquad (1.1)$$

with S_0 the basic action obtained by twisting an ordinary model, $\sigma_N(\phi^\alpha)$ the Nth gravitational descendant of ϕ^α, and $t_{N,\alpha}$ the coupling constants. It is essentially characterized by genus-zero 3-point functions $\langle \phi^\alpha \phi^\beta \phi^\gamma \rangle$. The topological field theory