Commun. Math. Phys. 182, 105 – 153 (1996)

Communications in Mathematical Physics
© Springer-Verlag 1996

On the Dimension of Deterministic and Random Cantor-like Sets, Symbolic Dynamics, and the Eckmann–Ruelle Conjecture

Yakov Pesin, Howard Weiss

Department of Mathematics, 310, McAlister Building, The Pennsylvania State University, University Park, PA 16802, USA. E-mails: pesin@math.psu.edu, weiss@math.psu.edu

Received: 15 November 1995 / Accepted: 14 Febrauary 1996

Abstract: In this paper we unify and extend many of the known results on the dimension of deterministic and random Cantor-like sets in $\mathbb{R}^n$, and apply these results to study some problems in dynamical systems. In particular, we verify the Eckmann–Ruelle Conjecture for equilibrium measures for Hölder continuous conformal expanding maps and conformal Axiom $A^{\#}$ (topologically hyperbolic) homeomorphims. We also construct a Hölder continuous Axiom $A^{\#}$ homeomorphism of positive topological entropy for which the unique measure of maximal entropy is ergodic and has different upper and lower pointwise dimensions almost everywhere. This example shows that the non-conformal Hölder continuous version of the Eckmann–Ruelle Conjecture is false.

The Cantor-like sets we consider are defined by geometric constructions of different types. The vast majority of geometric constructions studied in the literature are generated by a finite collection of p maps which are either contractions or similarities and are modeled by the full shift on p symbols (or at most a subshift of finite type). In this paper we consider much more general classes of geometric constructions: the placement of the basic sets at each step of the construction can be arbitrary, and they need not be disjoint. Moreover, our constructions are modeled by arbitrary symbolic dynamical systems. The importance of this is to reveal the close and nontrivial relations between the statistical mechanics (and especially the absence of phase transitions) of the symbolic dynamical system underlying the geometric construction and the dimension of its limit set. This has not been previously observed since no phase transitions can occur for subshifts of finite type.

We also consider nonstationary constructions, random constructions (determined by an arbitrary ergodic stationary distribution), and combinations of the above.

Introduction

In this paper we unify and extend many of the known results on the dimension of deterministic and random Cantor-like sets in $\mathbb{R}^n$. These sets are defined by geometric

The work of the first author was partially supported by a National Science Foundation grant #DMS91-02887. The work of the second author was partially supported by a National Science Foundation Post-doctoral Research Fellowship and National Science Foundation grant #DMS-9403724.