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Kauffman Bracket of Plane Curves

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Abstract: We lower the Kauffman bracket for links in a solid torus (see [16]) to generic plane fronts. It turns out that the bracket can be entirely defined in terms of a front itself without using the Legendrian lifting. We show that all the coefficients of the lowered bracket are in fact Vassilev type invariants of Arnold's J^+-theory [3, 4]. We calculate their weight systems. As a corollary we obtain that the first coefficient is essentially the quantum deformation of the Bennequin invariant introduced recently by M. Polyak [19].

There exists a straightforward way to get an invariant of an immersed cooriented hypersurface C in a smooth manifold N. We lift C to the manifold M of cooriented contact elements of N. This gives us an embedded submanifold L_C. Now we take the value of a known invariant of embeddings on $L_C \hookrightarrow M$ as the invariant of our initial immersion $C \looparrowright N$.

The manifold M of cooriented contact elements is the spherisation of the cotangent bundle of N: $M = ST^*N$. It has a natural contact structure. Our lifting L_C is a Legendrian submanifold with respect to this structure. The hypersurface C is called *the front* of L_C. The above procedure defines an invariant not only on immersed $C \looparrowright N$ but also on submanifolds with some "admissible" singularities which may appear as singularities of fronts of smooth Legendrian submanifolds generically embedded into M.

The simplest situation is $N = \mathbf{R}^2$. The "admissible" singularities in this case are cusps. Thus we can induce an invariant on collections of closed oriented and cooriented plane curves which may have only double points and cusps as singularities. The manifold M of contact elements of the plane is the solid torus $M = \mathbf{R}^2 \times S^1$. So the lifted submanifolds are Legendrian links in it. This general approach was used in [12] to define an invariant of an immersed plane curve. There a Kontsevich type integral [11] was taken as a known invariant of knots in a solid torus. In a

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