Commun. Math. Phys. 182, 33–82 (1996)

Communications in Mathematical Physics
© Springer-Verlag 1996

Localization Expansions. I. Functions of the "Background" Configurations

Tadeusz Balaban

Department of Mathematics, Rutgers University, New Brunswick, NJ 08903, USA

Received: 17 July 1995/Accepted: 2 January 1996

Abstract: Expansions of the type described in the inductive hypothesis (H.5) in the paper [1] are constructed for local functions of the "background" configurations, i.e., solutions of the variational problems studied in the previous paper [3]. A main part of this construction is a further analysis of a local structure of the solutions.

1. Introduction

In this paper we discuss a new kind of problem connected with expansions assumed in the inductive hypothesis (H.5) in [1], but methodologically it is a continuation of the paper [3] on the variational problems for background configurations. Our main concern here is to "localize" properly these configurations, and to do this we use extensively the results and methods of [3]. One of the most important technical problems in the renormalization group approach is to construct expansions of non-local functions of basic variables into sums of localized functions, like the expansions in (H.5) [1]. There are several types of non-localities and non-local functions, we have to consider. In this paper we construct such expansions for a simplest and most frequently occurring type of non-local functions given by compositions of localized functions with one of the background configurations, like for example terms $\mathscr{E}^{(j)}(X;\psi_k^{(j)})$ of the effective actions. Obviously values $\phi_k(x;\psi), \psi^{(j)}(y;\psi)$ at points x, y of corresponding lattices are non-local functions of ψ, in fact they depend on ψ on the whole lattice, or on a generating set $\mathbb{B}_k$ determining the functions.

Let us describe now in detail basic goals of this paper. Consider a generating set $\mathbb{B}_k$, which we can identify with the sequence of domains $\{\Omega_1,\dots,\Omega_k\}$, see the definitions (1.1)–(1.3) in [3], and assume that a next domain Ω_{k+1} is given, such that adding it to the sequence yields a new generating set $\mathbb{B}_{k+1}$. We denote $\Lambda = B(\Lambda_{k+1}) = \Omega_{k+1}^{(k)}$, i.e., $\Lambda \subset T^{(k)}$. Taking the first j domains in the sequence determines a generating set $\mathbb{B}_j$. Consider a function $\mathscr{E}(X;\psi_j)$, where X is a localization domain, $X \in \mathscr{D}_j$, and ψ_j is a spin variable on the lattice $T^{(j)}$. We assume that this function has the same properties as a term of the localization expansion in

This work has been partially supported by the NSF Grant DMS-9102639