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Higher Weil–Petersson Volumes of Moduli Spaces of Stable *n*-Pointed Curves

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Dedicated to the memory of Claude Itzykson

Abstract: Moduli spaces of compact stable n-pointed curves carry a hierarchy of cohomology classes of top dimension which generalize the Weil–Petersson volume forms and constitute a version of Mumford classes. We give various new formulas for the integrals of these forms and their generating functions.

0. Introduction

Let $\overline{M}_{g,n}$ be the moduli space of stable n-pointed curves of genus g. The intersection theory of these spaces is understood in the sense of orbifolds, or stacks. The algebro-geometric study of the Chow ring of $\overline{M}_{g,0}$ was initiated by D. Mumford.

The following important version of Mumford classes on $\overline{M}_{g,n}$ was introduced in [AC]. Let $p_n : \mathscr{C}_n \to \overline{M}_{g,n}$ be the universal curve, $x_i \subset \mathscr{C}_n$, $i = 1, \dots, n$, the images of the structure sections, $\omega_{\mathscr{C}/M}$ the relative dualizing sheaf. Put for $a \geqq 0$,

$$\omega_n(a) = \omega_{g,n}(a) := p_{n*}\left(c_1\left(\omega_{\mathscr{C}/M}\left(\sum_{i=1}^{n} x_i\right)\right)^{a+1}\right) \in H^{2a}(\overline{M}_{g,n}, \mathbb{Q})^{\mathbb{S}_n} . \qquad (0.1)$$

(We use here the notation of [KMK; AC] denote these classes κ_i. We will mostly omit g in our notation but not n).

The class $\omega_{g,n}(1)$ is actually $\frac{1}{2\pi^2}[v_{g,n}^{\mathrm{WP}}]$, where $v_{g,n}^{\mathrm{WP}}$ is the Weil–Petersson 2-form so that

$$\int_{\overline{M}_{g,n}} \omega_{g,n}(1)^{3g-3+n} = (2\pi^2)^{3g-3+n} \times \text{WP-volume of } \overline{M}_{g,n} . \qquad (0.2)$$

(see [AC], end of Sect. 1). Generally, we will call *higher* WP-*volumes* the integrals of the type

$$\int_{\overline{M}_{g,n}} \omega_{g,n}(1)^{m(1)} \cdots \omega_{g,n}(a)^{m(a)} \dots , \qquad \sum_{a \geqq 1} am(a) = 3g - 3 + n .$$

The objective of this paper is to derive several formulas for these volumes and their generating functions.