On Representations of the Elliptic Quantum Group $E_{\tau,\eta}(sl_2)$

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Abstract: We describe representation theory of the elliptic quantum group $E_{\tau,\eta}(sl_2)$. It turns out that the representation theory is parallel to the representation theory of the Yangian $Y(sl_2)$ and the quantum loop group $U_q(\widetilde{sl}_2)$.

We introduce basic notions of representation theory of the elliptic quantum group $E_{\tau,\eta}(sl_2)$ and construct three families of modules: evaluation modules, cyclic modules, one-dimensional modules. We show that under certain conditions any irreducible highest weight module of finite type is isomorphic to a tensor product of evaluation modules and a one-dimensional module. We describe fusion of finite dimensional evaluation modules. In particular, we show that under certain conditions the tensor product of two evaluation modules becomes reducible and contains an evaluation module, in this case the imbedding of the evaluation module into the tensor product is given in terms of elliptic binomial coefficients. We describe the determinant element of the elliptic quantum group. Representation theory becomes special if $N\eta = m + l\tau$, where N, m, l are integers. We indicate some new features in this case.

1. Introduction

The elliptic quantum group is an algebraic structure underlying the elliptic solutions of the Star–Triangle relation in statistical mechanics and connected with the Knizhnik–Zamolodchikov–Bernard equation on tori. In this paper we consider the elliptic quantum group $E_{\tau,\eta}(sl_2)$ defined in [Fe1-2] and discuss its representation theory. It turns out that representation theory of the elliptic quantum group $E_{\tau,\eta}(sl_2)$ is parallel to representation theory of the Yangian $Y(sl_2)$ and the quantum loop group $U_q(\widetilde{sl}_2)$ [T], cf. [CP].

We introduce basic notions of representation theory of the elliptic quantum group (notions of the operator algebra, a highest weight module, an irreducible module,