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Large Deviations and the Distribution of Pre-images of Rational Maps

Mark Pollicott, Richard Sharp

Department of Mathematics, University of Manchester, Oxford Road, Manchester M13 9PL, United Kingdom

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Abstract: In this article we prove a large deviation result for the pre-images of a point in the Julia set of a rational mapping of the Riemann sphere. As a corollary, we deduce a convergence result for certain weighted averages of orbital measures, generalizing a result of Lyubich.

0. Introduction

Let $\hat{\mathbb{C}}$ denote the Riemann sphere and let $T : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$ be a rational map of degree $d \geqq 2$, say. Every point has d pre-images (counted according to their multiplicities). There is a well-known result of Lyubich which shows that for a point $x \in \mathbb{J}$ in the Julia set an evenly distributed weight on the set of d^n pre-images

$$S_n(x) = \{y \in \hat{\mathbb{C}} : T^n y = x\}$$

converges (in the weak* topology) to a measure μ_0 as $n \to +\infty$ [4, 8]. The measure μ_0 is precisely the unique measure of maximal entropy for the map T [2, 5].

Since $T : \mathbb{J} \to \mathbb{J}$ is a continuous map on a compact metric space we can define the pressure of a continuous function $f : \mathbb{J} \to \mathbb{R}$ by

$$P(f) = \sup \left\{ h(v) + \int f dv \colon v \text{ is a } T\text{-invariant probability} \right\} ,$$

where $h(v)$ denotes the entropy of T with respect to v. An equilibrium state for f is a T-invariant probability μ realising this supremum.

Let $\mathscr{M}$ denote the set of all probability measures on $\mathbb{J}$. We shall show the following stronger "large deviation" result on the pre-images of a point $x \in \mathbb{J}$.

Theorem 1. *Let $f : \mathbb{J} \to \mathbb{R}$ be a Hölder continuous function such that $P(f) > \sup f$ and let μ be the unique equilibrium state for f. Let $x \in \mathbb{J}$. Then for any weak* open neighbourhood $\mathscr{U} \subset \mathscr{M}$ of μ we have that the weighted proportion of*

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