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Decay to Equilibrium in Random Spin Systems on a Lattice

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Abstract: We study continuous and discrete spin systems on a lattice with random interactions of finite range. In particular for sufficiently large interactions we prove no spectral gap property in the high temperature region. Moreover we show that in two dimensions, if the temperature is sufficiently high and the probability of interaction to be large is small enough, we have an almost sure stretched exponential upper bound for the decay to equilibrium.

1. Introduction

It is known that the spin systems with random interactions can exhibit a rather unusual behaviour even in the high temperature equilibrium, [21, 13]. This equilibrium behaviour has been extensively studied on the rigorous level, (see e.g. [5, 18, 7, 6, 25, 20, 14, 15, ..., 40] and references given there), and at the moment we have relatively good understanding of the high temperature phase. In particular it is known that for random spin systems with short range interactions with probability one we have an exponential decay of correlations with nonrandom characteristic length. The understanding of the corresponding stochastic dynamics on the rigorous level is much less advanced, although recently some progress has been made. In the direction of mean field models, the asymptotics of Langevin dynamics have been investigated in [2, 3, 4, 23] and have been proven to exhibit unusual non-Markovian features. For short range models, it has been shown in [39] that the generator of Glauber dynamics for the Ising type discrete spin systems on $\mathbb{Z}^d$ cannot have a spectral gap with probability one, if the couplings are allowed to take sufficiently large values. This is in distinction to nonrandom spin systems where the spectral gap is a typical feature of the high temperature phase, (see e.g. [1, 33–35] and references given there). The no spectral gap result of [39] has been strongly based on the finite volume estimates of the spectral gap in the phase transition region obtained in [37]. In the present paper we extend the result of [39] to systems where weaker information is available, including an interesting case of continuous spins. Such a

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