

# Adiabatic Curvature and the $S$ -Matrix

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**Abstract:** We study the relation of the adiabatic curvature associated to scattering states and the scattering matrix. We show that the curvature of the scattering states is not determined by the scattering data alone. However, for certain tight binding Hamiltonians, the Chern numbers are determined by the  $S$ -matrix and are given explicitly in terms of integrals of certain odd-dimensional forms constructed from the scattering data. Two examples, which are the natural scattering analogs of Berry's spin 1/2 magnetic Hamiltonian and its quadrupole generalization, serve to motivate the questions and to illustrate the results.

## I. Introduction

In this paper we study how the adiabatic curvature and Chern numbers of scattering states are related to the scattering matrix. One motivation comes from the theory of quantum transport where the adiabatic curvature, Chern numbers, and scattering data are all related to notions of conductance. (In the quantum Hall effect, the Hall conductance is related to a Chern number [13, 3]; in mesoscopic networks the charge transport is related to the adiabatic curvature [1]; the Landauer theory of quantum transport expresses the conductance in terms of scattering data [8].) Our aim is to study this chain of relations from a general perspective and without specific reference to quantum transport.

We shall consider local deformations of quantum Hamiltonians that are associated with a scattering situation and have a band of absolutely continuous spectrum. We study the adiabatic curvature associated with this band. We shall not consider deformations that “act at infinity”.

As we shall see, the  $S$ -matrix alone does not determine the adiabatic curvature. This may not be surprising, since even for potential scattering in one dimension the scattering matrix alone does not determine the scattering potential (one needs to know certain norming constants associated with bound states) [9]. On the other hand,

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