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Tail Estimates for One-Dimensional Random Walk in Random Environment

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Abstract: Suppose that the integers are assigned i.i.d. random variables $\{\omega_x\}$ (taking values in the unit interval), which serve as an environment. This environment defines a random walk $\{X_k\}$ (called a RWRE) which, when at x, moves one step to the right with probability ω_x, and one step to the left with probability $1 - \omega_x$. Solomon (1975) determined the almost-sure asymptotic speed (= rate of escape) of a RWRE. For certain environment distributions where the drifts $2\omega_x - 1$ can take both positive and negative values, we show that the chance of the RWRE deviating below this speed has a polynomial rate of decay, and determine the exponent in this power law; for environments which allow only positive and zero drifts, we show that these large-deviation probabilities decay like $\exp(-Cn^{1/3})$. This differs sharply from the rates derived by Greven and den-Hollander (1994) for large deviation probabilities conditioned on the environment. As a by product we also provide precise tail and moment estimates for the total population size in a Branching Process with Random Environment.

1. Introduction

In this paper we consider the large deviations of the position of a nearest-neighbor random walk on $\mathbb{Z}$ with site-dependent transition probabilities.

Let $\omega = (\omega_x)_{x\in\mathbb{Z}}$ be an i.i.d. collection of $(0,1)$-valued random variables, with marginal distribution α such that $\operatorname{supp}\alpha \subset (0,1)$. For every fixed ω, let $X = (X_n)_{n\geqq 0}$

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