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Entropic Repulsion of the Lattice Free Field, II. The 0-Boundary Case

Jean-Dominique Deuschel

Fachbereich Mathematik, Technische Universität Berlin, D-10623 Berlin, Germany.
E-mail: deuschel@math.tu-berlin.de

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Abstract: This paper is a continuation of [5]. We consider the Euclidean massless free field on a box V_N of volume N^d with 0-boundary condition; that is the centered Gaussian field with covariances given by the Green function of the simple random walk on $\mathbb{Z}^d, d \geqq 3$, killed as it exits V_N. We show that the probability, that all the spins are positive in the box V_N decays exponentially at a surface rate N^{d-1}. This is in contrast with the rate $N^{d-2}\log N$ for the infinite field of [5].

1. Introduction

The object of this paper is to analyze the asymptotical behavior of a Gibbsian Gaussian field, under the condition that the variables are positive in a large finite box. These asymptotics play an important role in the construction of droplets on a "hard surface", cf. [1, 6, 10], and in related questions dealing with quasi-locality, cf. [7], and entropic repulsion [7, 11].

More precisely, let $\Lambda = [-1,1]^d$ be the unit box in $\mathbb{R}^d$ and set $V_N = N\Lambda \cap \mathbb{Z}^d$. Next consider the Gaussian field P_N^0 on $\Omega_N = \mathbb{R}^{V_N}$ with density with respect to the Lebesgue measure $\lambda_N(dX) = \prod_{i\in V_N} dX(i)$ of the form

$$P_N^0(dX) = \frac{1}{Z_N}\exp\left(-\frac{1}{2}\sum_{\{i,j\}\cap V_N \neq \emptyset} Q_{\mathrm{d}}(i,j)(X(i)-X(j))^2\right)\lambda_N(dX)\,, \tag{1.1}$$

where Z_N is a normalizing constant, $Q_{\mathrm{d}}(i,j) = \frac{1}{2d}1_{|i-j|=1}$ is the transition matrix of the simple random walk on $\mathbb{Z}^d$, and we set $X(j) = 0$ for $j \notin V_N$. Thus the spins are "tied down" at the boundary of V_N. P_N^0 can be viewed as the finite Gibbs distribution on Ω_N to the nearest neighbor quadratic interaction

$$\mathscr{J} = \{J_{\{i,j\}}(X) = Q_{\mathrm{d}}(i,j)(X(i)-X(j))^2,\ \{i,j\} \subseteq \mathbb{Z}^d\}$$

with 0-boundary conditions on $V_N^{\complement}$. We will be working in the transient dimensions $d \geqq 3$; then P_N^0 converges weakly to P^0, the infinite Gibbs distribution, sometimes called (discrete) *Euclidean massless free field.* P^0 is the centered Gaussian field on