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Solutions of Klein–Gordon and Dirac Equations on Quantum Minkowski Spaces

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Abstract: Covariant differential calculi and exterior algebras on quantum homogeneous spaces endowed with the action of inhomogeneous quantum groups are classified. In the case of quantum Minkowski spaces they have the same dimensions as in the classical case. Formal solutions of the corresponding Klein–Gordon and Dirac equations are found. The Fock space construction is sketched.

0. Introduction

It is well known that lattice-like theories serve as regularization schemes in quantum field theory. But after introducing the lattice, we no longer have the full symmetry of the original theory. On the other hand, there was a lot of interest in quantum spacetimes endowed with the actions of quantum groups which are deformations of the objects used in the standard field theory (cf. [20, 3, 8, 7, 24, 13, 9, 5, 29, 19]). There were two motivations of such a development: providing naive models of changed geometry at the Planck scale and attempts to regularize the theory while preserving the "size" of the symmetry group in such a way that the regularized theory could still be imagined as the theory of our universe. Although the present paper doesn't provide support for any of these claims, we find a lattice-like behavior of certain quantum Minkowski spaces. It has two aspects:

1. It was found [12] that in the differential calculus on $\mathbf{R}$ corresponding to the one-dimensional lattice one has

$$xdx = (dx)x + ldx\,,$$

where x is the identity function and l is the lattice constant. In Sect. 1 we describe differential calculi on quantum Minkowski spaces by a very similar relation (1.7).

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