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Drinfeld–Sokolov Gravity

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Abstract: A lagrangian euclidean model of Drinfeld–Sokolov (DS) reduction leading to general W-algebras on a Riemann surface of any genus is presented. The background geometry is given by the DS principal bundle K associated to a complex Lie group G and an $SL(2,\mathbb{C})$ subgroup S. The basic fields are a hermitian fiber metric H of K and a $(0,1)$ Koszul gauge field A^* of K valued in a certain negative graded subalgebra $\mathfrak{x}$ of $\mathfrak{g}$ related to $\mathfrak{s}$. The action governing the H and A^* dynamics is the effective action of a DS field theory in the geometric background specified by H and A^*. Quantization of H and A^* implements on one hand the DS reduction and on the other defines a novel model of $2d$ gravity, DS gravity. The gauge fixing of the DS gauge symmetry yields an integration on a moduli space of DS gauge equivalence classes of A^* configurations, the DS moduli space. The model has a residual gauge symmetry associated to the DS gauge transformations leaving a given field A^* invariant. This is the DS counterpart of conformal symmetry. Conformal invariance and certain non-perturbative features of the model are discussed in detail.

1. Introduction

In recent years, a considerable amount of work has been devoted to the study of W-algebras [1]. The interest in W-algebras stems mainly from the fact that they are non-linear extensions of the Virasoro algebra appearing as symmetry algebras in certain critical two dimensional statistical systems as well as in W strings and W-gravity models. The latter in turn are of considerable interest in themselves as generalizations of ordinary string and gravity models with non-standard values of the critical dimension [2–5].

The construction of W-algebras can be carried out both in a hamiltonian and in a lagrangian framework. In the former approach [6–12], based on the methods of hamiltonian reduction, the currents of a Wess–Zumino–Novikov–Witten phase space with the standard Kac–Moody Poisson structure and Virasoro action are subject to a set of conformally invariant first class constraints corresponding to a certain nilpotent subalgebra of the relevant symmetry Lie algebra. Upon gauge fixing, the reduced phase space exhibits a non-linear Poisson structure and a Virasoro action, realizing