Commun. Math. Phys. 181, 507–528 (1996)

Communications in Mathematical Physics

Phase Transition in Continuum Potts Models

H.-O. Georgii[1,*], **O. Häggström**[2,**]

[1] Mathematisches Institut der Universität München, Theresienstr 39, D–80333 München, Germany.
E-mail: georgii@rz.mathematik.uni–muenchen.de
[2] Dept of Mathematics, Chalmers University of Technology, 412 96 Göteborg, Sweden
E-mail: olleh@math chalmers.se

Received: 25 February 1996 / Accepted: 21 April 1996

To the memory of Roland Dobrushin

Abstract: We establish phase transitions for a class of continuum multi–type particle systems with finite range repulsive pair interaction between particles of different type. This proves an old conjecture of Lebowitz and Lieb. A phase transition still occurs when we allow a background pair interaction (between all particles) which is superstable and has sufficiently short range of repulsion. Our approach involves a random–cluster representation analogous to the Fortuin–Kasteleyn representation of the Potts model. In the course of our argument, we establish the existence of a percolation transition for Gibbsian particle systems with random edges between the particles, and also give an alternative proof for the existence of Gibbs measures with superstable interaction.

1. Introduction

Although the study of phase transitions for Gibbsian systems is one of the main subjects of statistical mechanics, examples of models exhibiting phase transition are mainly restricted to lattice systems. For systems of particles in the continuum the situation is still quite unsatisfactory. Besides the mere existence of phase transitions for some unknown interactions (as shown in Appendix B of Israel [17]) and the canonical–ensemble approach of Johansson [18, 19] in one dimension, there exists essentially only one specific model for which a phase transition is known to occur: the model of Widom and Rowlinson [32]. This is a multi–type particle system in $\mathbb{R}^d$, $d \geq 2$, with hard-core exclusion between particles of different type, and no interaction between particles of the same type. The phase transition in this model for large activities z was established by Ruelle [31] using a version of the Peierls argument. Lebowitz and Lieb [23] extended his result to multi–type particle systems in which the hard–core exclusion is replaced by a soft–core repulsion between unlike particles. Such models can also be viewed as continuum versions of the Ising or Potts model. Lebowitz and Lieb needed the condition that the soft–core repulsion

* Research partially supported by the Isaac Newton Institute Cambridge.
** Research supported by the Swedish Natural Science Research Council and the Deutsche Forschungsgemeinschaft