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A Necessary and Sufficient Condition for the Existence of Multisolitons in a Self-Dual Gauged Sigma Model

Yisong Yang★

Department of Applied Mathematics and Physics, Polytechnic University, Brooklyn, New York 11201, USA

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Abstract: This paper presents a resolution of the gauged $O(3)$ sigma model proposed by B.J. Schroers in which the matter field ϕ maps $\mathbf{R}^2$ into S^2 while the vector gauge potential gives rise to a magnetic field. It is shown that for each natural number N there are solutions to saturate the classical energy lower bound $E \geqq 4\pi N$ for the field configurations in the topological family $\deg(\phi) = N$ if and only if $N \neq 1$. Furthermore the solutions obtained depend on at least $4N - 3$ continuous parameters, the associated magnetic flux can assume its value in an open interval, and the decay rates of the field strengths may be specified in a suitable range. These solutions are multisolitons represented by N prescribed lumps of the magnetic field, simulating N identical particles in equilibrium, and are governed by a nonlinear elliptic equation with both vortex and anti-vortex source terms.

1. Introduction and Main Results

In this paper we are interested in static solutions of a gauged $O(3)$ sigma model which originates from the classical planar ferromagnet model defined by the energy functional

$$E(\phi) = \frac{1}{2} \int (\partial_1 \phi)^2 + (\partial_2 \phi)^2 \,, \tag{1}$$

where the spin vector $\phi = (\phi_1, \phi_2, \phi_3)$ maps $\mathbf{R}^2$ into S^2, namely, $\phi_1^2 + \phi_2^2 + \phi_3^2 = 1$, and the integral is taken over the full $\mathbf{R}^2$ under the Lebesgue measure d^2x (unless otherwise stated). Finite energy condition implies that ϕ goes to a constant unit vector, say ϕ_∞, at infinity, which makes ϕ a continuous map from S^2 to S^2 so that the Hopf degree, $\deg(\phi)$, is well defined. In fact the work of Belavin and Polyakov [1, 2] establishes that the energy (1) has the topological lower bound

$$E(\phi) \geqq 4\pi |\deg(\phi)| \,, \tag{2}$$

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