A Variational Problem for a System of Magnetic Monopoles Joined by Abrikosov Vortices

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Abstract: An action functional, related to the Higgs model to field theory, depending on a complex scalar field and a $U(1)$ connection is defined. The complex scalar field is a section of a line bundle associated to a principal $U(1)$-bundle with base space $\mathbb{R}^3 \backslash \{x_1, \ldots, x_n\}$. The points $x_1, \ldots, x_n$ are the positions of n magnetic monopoles of magnetic charges $m_1, \ldots, m_n$, with $\sum_{i=1}^{n} m_i = 0$. The existence of minimizers of the action functional is proven using direct methods of the calculus of variation. Regularity and decay properties of the minimizers are obtained. By constructing explicit comparison field configurations, we establish accurate upper and lower bounds for the action of the minimizers in a variety of special situations, e.g. $n = 2$ and $m_1 = -m_2$.

Contents

1. Introduction

The variational problem studied in this paper arises in the description of the quantum counterparts of classical vortex configurations in the $U(1)$-Higgs model in $2 + 1$ space-time dimensions. Using Euclidean functional integral methods to construct the Green functions of the $U(1)$-Higgs model one is led to study the classical variational problem described in the abstract: In attempting to calculate these Green