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Low Temperature Phase Diagrams for Quantum Perturbations of Classical Spin Systems

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Abstract: We consider a quantum spin system with Hamiltonian

$$H = H^{(0)} + \lambda V,$$

where $H^{(0)}$ is diagonal in a basis $|s\rangle = \bigotimes_x |s_x\rangle$ which may be labeled by the configurations $s = \{s_x\}$ of a suitable classical spin system on $\mathbb{Z}^d$,

$$H^{(0)}|s\rangle = H^{(0)}(s)|s\rangle.$$

We assume that $H^{(0)}(s)$ is a finite range Hamiltonian with finitely many ground states and a suitable Peierls condition for excitations, while V is a finite range or exponentially decaying quantum perturbation. Mapping the d dimensional quantum system onto a *classical* contour system on a $d+1$ dimensional lattice, we use standard Pirogov–Sinai theory to show that the low temperature phase diagram of the quantum spin system is a small perturbation of the zero temperature phase diagram of the classical Hamiltonian $H^{(0)}$, provided λ is sufficiently small. Our method can be applied to bosonic systems without substantial change. The extension to fermionic systems will be discussed in a subsequent paper.

1. Introduction

1.1. General ideas. Many models of classical statistical mechanics provide examples of first-order phase transitions and phase coexistence at low temperatures. It became clear already from the first proof of such a transition for the Ising model by the Peierls argument [Pei36, Gri64, Dob65] that a convenient tool for the study of phase coexistence and first-order phase transitions is a representation in terms of configurations of geometrical objects – contours. This has been systematically

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